

Mark Scheme (Results)

January 2016

International GCSE Further Pure Mathematics 4PM0/02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners
 must mark the first candidate in exactly the same way as they
 mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Types of mark
 - o M marks: method marks
 - o A marks: accuracy marks
 - B marks: unconditional accuracy marks (independent of M marks)
- Abbreviations
 - cao correct answer only
 - o ft follow through
 - o isw ignore subsequent working
 - o SC special case
 - o oe or equivalent (and appropriate)
 - o dep dependent
 - o indep independent
 - o eeoo each error or omission

- No working
 If no working is shown then correct answers may score full marks
 If no working is shown then incorrect (even though nearly correct)
 answers score no marks.
- With working Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.

If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

- Ignoring subsequent work
 It is appropriate to ignore subsequent work when the additional
 work does not change the answer in a way that is inappropriate for
 the question
- Parts of questions
 Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking (but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q) \text{where} |pq| = |c|$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{where} |pq| = |c| \text{ and } |mn| = |a|$$

2. Formula:

Attempt to use the correct formula (shown explicitly or implied by working) with values for a, b and c, leading to

3. Completing the square:

Solving
$$x^2 + bx + c = \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$$
 where $q \neq 0$

Method marks for differentiation and integration:

1. <u>Differentiation</u>

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

Use of a formula:

Generally, the method mark is gained by:

either quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is <u>not</u> quoted, the method mark can be gained by implication from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.



Jan 2016

4PM0 Further Pure Mathematics Paper 2

Mark Scheme

Question Number	Scheme	Marks
1.	$2^{2(x-2)} = 2^{3(3x-1)}$ $\Rightarrow 2(x-2) = 3(3x-1)$ $x = -\frac{1}{7}$	M1 dM1A1 A1cao (4)
M1 dM1 A1 A1cao	Attempt to change to powers of 2, 4 or 8 (both sides of equation) Equate powers Correct linear equation - unsimplified $x = -\frac{1}{7}$ (or equivalent fraction with integer numerator and denomination NB: $\log_4 8 = 1.5$ is exact and so allowed	ator)
ALT 1	Alternatives for no 1 Take logs base 4 each side Change log48 to 1.5 Correct linear equation 1.5 and any other non-rounded decimals allo $x = -\frac{1}{7}$ Correct solution 7 decimals may have been used in working, properties to the control of the c	
ALT 2	$\log 4^{(x-2)} = \log 8^{(3x-1)}$ can be any base $(x-2)\log 4 = (3x-1)\log 8$ $(x-2)\times 2\log 2 = (3x-1)\times 3\log 2$	M1
	$(x-2) \times 2\log 2 = (3x-1) \times 3\log 2$ $2(x-2) = 3(3x-1)$ $x = -\frac{1}{7}$	dM1A1 A1cao

Question Number	Scheme	Marks
	$\frac{4^{x}}{4^{2}} = \frac{8^{3x}}{8} \Rightarrow \frac{4^{x}}{2} = 8^{3x}$ $4^{x} \times \frac{1}{2} = \left(8^{3}\right)^{x} \qquad \frac{1}{2} = \left(\frac{8^{3}}{4}\right)^{x}$	M1
	$\frac{1}{2} = 128^{x}$ $x = \frac{\log \frac{1}{2}}{\log 128} = \frac{-\log 2}{7 \log 2} \text{(any base)}$	dM1A1
	1	A1cao
2.	(i) $48 = \frac{1}{2}\theta r^2$, $8 = \theta r$ or equivalent equations $\frac{\theta r^2}{\frac{2}{\theta r}} = \frac{48}{8} \Rightarrow r = 12$ (ii) $\theta = \frac{8}{12}, (=\frac{2}{3})$	B1B1 M1A1 A1 (5)
B1 B1 M1 A1 A1	B1B1 Two correct equations; B1B0 One correct equation Eliminate either variable and solve to obtain the other $r = 12$ $\theta = \frac{8}{12}$ oe Accept 0.667 or better (NB: decimal may be ignored rule.)	under isw

Question Number	Scheme						
3	$3y = 12 - 4x \Rightarrow y = 4 - \frac{4}{3}x$ OR $4x = 12 - 3y \Rightarrow x = 3 - \frac{3}{4}y$						
	$\left[(x+1)^2 + (4 - \frac{4}{3}x - 2)^2 = 4 \right] \left[\left(3 - \frac{3}{4}y + 1 \right)^2 + (y-2)^2 = 4 \right]$	M1					
	$\Rightarrow 25x^2 - 30x + 9 = 0 \text{ 3TQ} $ $\Rightarrow 25y^2 - 160y + 256 = 0 \text{ 3TQ}$	M1A1					
	$(5x-3)(5x-3) = 0 \Rightarrow x = \frac{3}{5} \qquad (5y-16)(5y-16) = 0 \Rightarrow y = \frac{16}{5}$	M1A1					
	$y = 4 - \frac{4}{3} \times \frac{3}{5} = \frac{16}{5}$ $x = 3 - \frac{3}{4} \times \frac{16}{5} = \frac{3}{5}$	A1 (7)					
B1	Write the linear equation to read $x =$ or $y =$ May be seen explicit implied by subsequent working. (Equivalent forms accepted)	tly or					
M1	Substitute to obtain a quad equation in one variable						
M1	Simplify to a 3 term quadratic - terms in any order - coeffs need not be	e integers					
A1	Correct 3 term quadratic - terms in any order - coeffs need not be inte	egers					
M1	Their 3 term quadratic solved by any valid method. (Can still be earned if the discriminant is negative.)						
A1	Correct values for one variable						
A1	(B1 on e-pen) Correct values for the second variable Equivalents accepted for both variables NB: Calculator solutions for the quadratic accepted provided both roots correct.						
4							
	$f'(x) = 2e^{2x}(x+1)^{0.5} + e^{2x} \frac{(x+1)^{-0.5}}{2}$ M1A1A1						
	$f'(x) = e^{2x} \left(2(x+1)^{0.5} + \frac{1}{2(x+1)^{0.5}} \right)$ dM1						
	$\Rightarrow e^{2x} \left(\frac{4(x+1)+1}{2(x+1)^{0.5}} \right) \Rightarrow \frac{e^{2x}(4x+5)}{2\sqrt{x+1}} ***$	dM1A1cso (6)					
M1	Attempt to differentiate using the product rule. Must be the sum of two terms both with $(x + 1)^{+/-0.5}$ and e^{2x} . Constants may be incorrect						
	If quotient rule is used the numerator must be the difference of two te	erms both					
A 1 A 1	with $(x + 1)^{+/-0.5}$ and e^{2x} and the denominator must be $(x + 1)^{-1}$.						
A1A1 dM1	A1A1 Both terms fully correct; A1A0 one term fully correct						
	Extract a common factor of form $k e^{2x}$ where k is an integer						
dM1	Simplify the bracket by combining to a single term The above steps may be carried out in either order but marks must be entered in this order. These 2 M marks are dependent on the first M mark but not on each other.						
Alcso	Obtain the GIVEN answer with no errors seen $(x+1)^{\frac{1}{2}}$ scores A0						

Question	Scheme	Marks				
Number						
5 (a)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \Rightarrow \alpha\beta = \frac{5^2 - 19}{2} = 3 \text{ cso } ***$					
(b)	$\Rightarrow \frac{c}{a} = 3 \text{ and } -\frac{b}{a} = 5 \text{ let } a = 1 \Rightarrow x^2 - 5x + 3 = 0 \text{ oe}$	M1A1 (2)				
(c)	$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{\alpha^2 + \beta^2}{\alpha \beta}, = \frac{19}{3}$	M1,A1				
		WII,AI				
	$\frac{\beta}{\alpha} \times \frac{\alpha}{\beta} = 1$	B1				
	$x^{2} - \frac{19}{3}x + 1 = 0$, $3x^{2} - 19x + 3 = 0$ oe	M1,A1 (5) (9)				
(a)M1	Obtain an expression for $\alpha\beta$ in terms of $\alpha + \beta$ and $\alpha^2 + \beta^2$					
A1cso	Correct value for $\alpha\beta$					
ALT:	Solve the given equations for α and β M1 Fully correct to given a	nswer A1				
(b)M1	Use $x^2 - (\text{sum of roots})x + \text{product of roots} (=0)$					
A1	A correct equation - any integer multiple of the one shown					
(c)M1	Write the sum of the roots as a single fraction. Algebra to be correct for the	is mark.				
A1	Correct value for the sum of the roots					
B1 M1	Product = 1 Seen explicitly or used					
	Use $x^2 - (\text{sum of roots})x + \text{product of roots} (=0)$					
A1ft	Correct equation. Follow through their sum and product. Any integer accepted.	multiple				
6 (a)	$\sin(2x) = \sin x \cos x + \cos x \sin x = 2\sin x \cos x *$	B1				
(b)	$\cos(2x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x *$	B1 (2)				
(c)	$\sin 2x$ $2\sin x \cos x$	3.41				
	$\frac{\sin 2x}{1+\cos 2x} = \frac{2\sin x \cos x}{1+(\cos^2 x - \sin^2 x)}$	M1				
	$2\sin x\cos x$	dM1A1				
	$= \frac{1}{\cos^2 x + \sin^2 x + \cos^2 x - \sin^2 x}$					
	$= \frac{2\sin x \cos x}{2\cos^2 x} = \tan x ***$	A1cso (4) (6)				
(a)B1	For the correct result. Award only if evidence of use of the given form	nula is seen				
(b)B1	As for (a)					
(c)M1	Use the above identities to change "2x"s to "x"s					
dM1	Use $\cos^2 x + \sin^2 x = 1$ to eliminate $\sin^2 x$					
	Min evidence is $(1-\sin^2 x)$ changed to $\cos^2 x$ or $(1-\sin^2 x) + \cos^2 x$	$=2\cos^2 x$				
	Denominator $1 + c^2 - s^2$ changed to either c^2+c^2 or $2c^2$ is NOT sufficient	ent				
	But $1 - s^2 + c^2$ changed to $c^2 + c^2$ or $2c^2$ is sufficient					
A1	Correct (unsimplified) fraction, as shown or equivalent (no trig functions of $2x$)					
Λ1	Both M marks must be gained for this A mark to be awarded					
A1cso	Obtain the GIVEN result with no errors seen					

Question Number	Scheme	Marks				
7 (a)	$x = \frac{3}{2}$ (or eg $2x = 3$, $x - \frac{3}{2} = 0$)	B1 (1)				
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-3)(2x) - (x^2 - 2)(2)}{(2x-3)^2} = \left(\frac{2x^2 - 6x + 4}{(2x-3)^2}\right)$	M1A1A1 (3)				
(c)	$\frac{dy}{dx} = 0 \Rightarrow \frac{(2x-3)(2x) - (x^2 - 2)(2)}{(2x-3)^2} = 0$	M1				
	$\Rightarrow 2x^2 - 6x + 4 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow x = 1, x = 2$	M1A1A1				
	x = 1, y = 1 (1,1) $x = 2, y = 2$ (2,2)	A1 (5) (9)				
(a) B1	For a correct equation for the asymptote. NB $x \neq \frac{3}{2}$ scores B0					
(b) M1 A1 A1	Attempt to differentiate by quotient rule. Denominator must be correct. Numerator must be the difference of two terms of the appropriate form. NB M1 on e-PEN First term correct					
ALT:	Second term correct Use the product rule. M1 for the attempt, using $(x^2-2)(2x-3)^{-1}$					
(c) M1	A1,A1 one for each correct term Equate their derivative to 0					
M1 A1 A1	Solve their quadratic (numerator) by any valid method. A1A1 two correct values for <i>x</i> from a correct equation; A1A0 for one correct value Ignore extra values.					
A1	NB B1 on e-PEN Find the corresponding y values. Coordinate brackets need not be shown. Give A0 if more than 2 stationary points shown.					
	NB: Quadratic solved on a calculator: correct values for <i>x</i> , M1A1A1 One or both values incorrect, or only one value shown: M0A0A0					
	Special Case for (c): Both c orrect answers only shown, Award B1B two marks on e-PEN.	1 - in first				

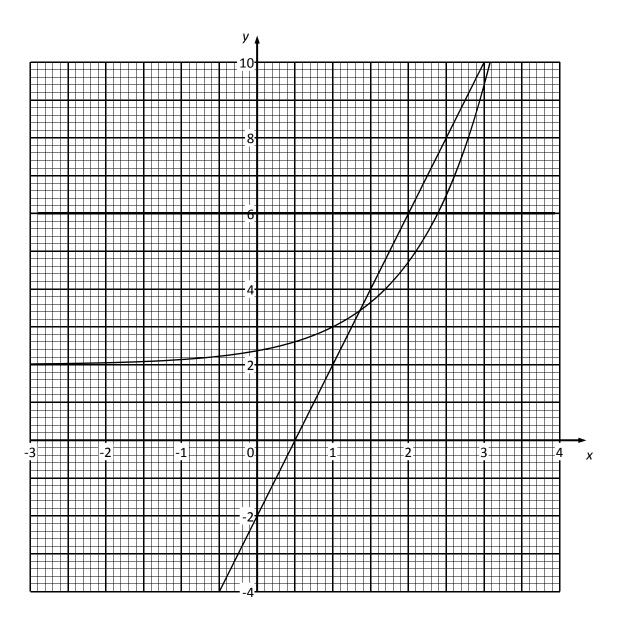
Question	Scheme	Marks			
Number					
8 (a)	$a = 2 - 3 = -1 \ d = 2 \qquad (l = 2n - 3)$ $Uses S_n = \frac{n}{2}(a + l), S_n = \frac{n}{2}(-1 + (2n - 3)) S_n = \frac{n}{2}(n - 2) ***$	B1B1			
	OR $S_{n} = \frac{n}{2}(2 \times -1 + (n-1)2) \Rightarrow S_{n} = \frac{n}{2}(2n-4) \Rightarrow S_{n} = n(n-2) ***$	M1A1cso			
(b)	5(2n+4-3) = 3(n-3)((n-3)-2)	(4)			
	3(2n+1)(n-3)(n-3)(2)	M1A1			
	$3n^2 - 34n + 40 = 0$ 3TQ $\Rightarrow (3n - 4)(n - 10) = 0 \Rightarrow n = 10$	M1 dM1A1			
		(5)			
(a) B1 B1	$a=-1$ No working needed - need not be shown explicitly $d=2$ No working needed or if $S_n=\frac{n}{2}(a+l)$ used, give B1 for correct subvalue shown anywhere for d	ostitution if no			
M1 A1cso (b)	Using either formula for S_n with their a and d Obtaining the GIVEN result with no errors seen				
M1 A1	Using the GIVEN t_n and S_n in the equation or start from correct basic formulae Correct unsimplified equation				
M1 dM1	Obtaining a three term quadratic, terms in any order NB A1 on e-pen Factorising their quadratic or correct use of formula/completing the				
A1	square. Cao $n = 10$ Award A0 if single correct answer not identified. If final answers shown without working (implying calculator solution) give M1 only if both correct answers to the quadratic are shown. A1 then for identifying the single correct solution for this problem.				

Question Number	Scheme	Marks			
9 (a)	$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} \Rightarrow \overrightarrow{OC} = 2\mathbf{b} - 2\mathbf{a} = 2(\mathbf{b} - \mathbf{a}) (= 2\overrightarrow{AB}) \text{ (oe)}$ (i) Hence, \overrightarrow{OC} and \overrightarrow{AB} are in same direction (ii) And, \overrightarrow{OC} is twice the length of \overrightarrow{AB} Conclusions required *	B1 M1, A1 A1 (4)			
(b)	$\frac{\text{area of triangle } ODC}{\text{area of triangle } OBC} = \frac{0.5 \times \text{height} \times 2}{0.5 \times \text{height} \times 5} = \frac{2}{5}$	M1A1			
	$\frac{\text{area of triangle } OAB}{\text{area of triangle } OBC} = \frac{0.5 \times \text{height} \times 1}{0.5 \times \text{height} \times 2} = \frac{1}{2}$ $\text{area of triangle } OBC = \frac{5}{2} \times \text{area of triangle } ODC \text{ , and,}$	M1A1			
	area of triangle $OBC = 2 \times \text{area of triangle } OAB$				
	Therefore, $\frac{\text{area of triangle }ODC}{\text{area of triangle }OAB} = \frac{4}{5}$ {Or given as ratio, area of triangle ODC ; area of triangle $OAB = 4:5$ }	dM1A1cso (6)			
(a) B1 M1 (i)A1 (ii)A1 (b) M1 A1 M1	Correct expression for \overline{AB} Obtaining \overline{OC} in terms of \mathbf{a} and \mathbf{b} Using correct expressions for \overline{OC} and \overline{AB} to deduce that they are parallel NB B1 on e-PEN Deducing the GIVEN ratio $AB:OC$ or $OC:AB$ provided clear which is intended. No vector arrows here. Accept shown or $\#$ or similar as a conclusion provided clear which part it refers to. Finding the ratio of the areas of triangles ODC and OBC , either order Correct ratio (or fraction), triangles in either order Finding the ratio of the areas of triangles OAB and OBC , either order				
A1 dM1 A1cso	Correct ratio (or fraction), triangles in either order Eliminating area of triangle <i>OBC</i> to obtain a value for the required ratio (Depends on both the preceding M marks. Correct ratio or fraction (any equivalent). Triangles to be in the correct of Ratio can be in one of forms 1:1.25, 1:5/4, 0.8: 1, 4/5:1				
	NB : b - a (whether bold, underlined or neither) is a vector, not the length M marks only can be awarded.	of a line.			

	Alternatives for 9(b)	
ALT 1	Area $\triangle OAB = \frac{1}{2}AB \times OB \sin OBA$	M1 (area either triangle)
	Area $\triangle ODC = \frac{1}{2}OD \times OC \sin DOC$	A1 (both areas correct)
	$2 \overrightarrow{AB} = \overrightarrow{OC} \text{ or } 2AB = OC, \qquad \frac{2}{5} \overrightarrow{OB} = \overrightarrow{OD} $	M1 (either)
	$\angle OBA = \angle DOC$ correct or used correctly)	A1 (all 3 statements
	$\therefore \triangle ODC : \triangle OAB = \left(\frac{1}{2}\right)AB \times OB : \left(\frac{1}{2}\right) \times 2AB \times \frac{2}{5}OB$	dM1 (their ratio of lengths)
	=4:5	A1
ALT 2	If $\frac{1}{2} \times \text{base} \times \text{height used}$:	
	Area $\triangle OAB = \frac{1}{2}AB \times h$	M1
	Area $\triangle ODC = \frac{1}{2}OC \times h'$	A1
	$h' = \frac{2}{5}h \ OC = 2AB$	M1A1
	$\triangle OCD : \triangle OAB = AB \times \frac{2}{5}h : \frac{1}{2}AB \times h dM1$	
	=4:5 oe	A1
	M1A1 areas of triangles (M1 either correct, A1 b M1A1 ratio of bases and ratio of heights (M1 either correct) dM1A1 correct completion	,

Question Number	Scheme	Marks
10 (a)	$f(2) = 2 \times 2^{3} - p \times 2^{2} - 13 \times 2 - q = -20 (\Rightarrow 10 = 4p + q)$ $f(3) = 2 \times 3^{3} - p \times 3^{2} - 13 \times 3 - q = 0 (\Rightarrow 15 = 9p + q)$	M1A1 M1A1
(A)	Solves simultaneous equations by elimination or substitution; $\Rightarrow 5 = 5p \Rightarrow p = 1$, so $q = 6$	M1 A1 A1 (7)
(b)	$(2x^{3} - x^{2} - 13x - 6) \div (x - 3) = 2x^{2} + 5x + 2$ $(2x^{3} - x^{2} - 13x - 6) = (x - 3)(2x + 1)(x + 2) \text{ (Factorises } 2x^{2} + 5x + 2)$ $x = 3, -\frac{1}{2}, -2 \text{ (all three roots)}$	M1A1 M1 A1A1 (5) (12)
(a) M1 A1 M1 A1	Substitute ± 2 in $f(x)$ Correct equation using remainder -20 Need not be simplified Substitute ± 3 in $f(x)$ Correct equation using remainder 0 Need not be simplified First 4 marks can be given for long division: Divide by $(x\pm 2)$ M1 Equate correct remainder to -20 A1 Divide by $(x\pm 3)$ M1 Equate correct remainder to 0 A1	
M1 A1 A1 (b) M1	Solve the simultaneous equations, any valid method p or q correct Second unknown correct Obtain the quadratic factor by division or inspection. Factor need not be correct but must be of form $2x^2 + kx \pm \frac{\text{their } q}{3}$ If by division, remainded be 0.	e fully r need not
A1 M1 A1A1	Correct quadratic factor Attempt to factorise their quadratic factor A1A1 all three roots correct; A1A0 two roots correct	

Question Number				Schen	ne			Marks
11(a)	x	-2	-1	0	1	2	3	B1B1 (2)
	f(x)	2.05	2.14	2.37	3	4.72	9.39	(2)
(b)	Correct po	ints plot	ted and g	graph draw	'n			B1ftB1ft
(c)	$4 = e^{(x-1)} = $ Line $y = 6$		•					(2) M1 A1 (2)
(d)	$\ln(4x-4)$ $\Rightarrow 4x-2 = y = 4x-2$ accept $x = $	= e ^(x-1) + drawn c	2	$)=e^{(x-1)},$				M1,A1 A1ft dM1 A1cso(5) (11)
(a) B1B1	NB Read r B1B1 thre	_						
(b) B1ft B1ft (c) M1	points/grap	ooth cur oh outsic deduce	ve throu le this rai	nge.			•	- ignore any on, $y = 4 \pm 2$
A1	Using $y = 6$ to obtain $x = 2.4$ Must be 1 dp unless already penalised (2.3862) If the M mark is gained and $y = 6$ or $e^{(x-1)} + 2 = 6$ is seen this mark can be given without the line being drawn. If the line $y = 6$ is seen on the graph and correct answer given, award M1A1							
(d) M1 A1 A1ft dM1 A1cso	Change eq Correct ex Add 2 to e Draw their	uation from the ponential ach side line on a 1.3 or om income	om log t d equation of their of their gray 1.4 Mustrect line	o exponents on equation ph st be 1 dp s score A0	tial formunities to the second	m already pen		55) Correct



Question Number	Scheme	Marks				
12(a)	$BM = \sqrt{8^2 - 4^2}, = 4\sqrt{3} \text{ (oe eg } \sqrt{24} \times \sqrt{2} \text{)}$	M1,A1A1 (3)				
(b) (c)	$p = 4 q = 3$ $\cos BAM = \frac{4}{8} \Rightarrow BAM = 60^{\circ}$ $EM = \sqrt{12^2 + 20^2} \left(= \sqrt{544} = 4\sqrt{34} \right)$	M1A1 (2) M1A1				
	$EM = \sqrt{12} + 20 (= \sqrt{344} = 4\sqrt{34})$ $MEB = \tan^{-1} \left(\frac{4\sqrt{3}}{4\sqrt{34}} \right) = 16.5437 \Rightarrow MEB = 16.5^{\circ}$	dM1A1(4)				
(d)	Angle between plane <i>BCEH</i> and <i>ADEH</i> = $\tan^{-1} \left[\frac{4\sqrt{3}}{20} \right] = 19.1066 = 19.1^{\circ}$	M1 dM1A1 (3) (12)				
(a) M1 A1 A1	Use Pythagoras Must have minus sign A1A1 for correct p and q equivalent values allowed as long as on A1A0 for one correct. Values need not be shown explicitly.					
(b) M1	Use any trig function correctly (eg sin = $\frac{\text{opp}}{\text{hyp}}$) to find $\angle BAM$ If cos or tan used then AM must = 4 or working for length AM must be seen. Their BM if used					
A1	Correct answer. 60° without working scores M1A1					
(c) M1	Use Pythagoras to find length <i>EM</i> . Must have + sign. If <i>BE</i> found without first finding <i>EM</i> this mark requires a complete method. Award M1 for $EM^2 = 16^2 + 20^2$ provided this is stated to be <i>EM</i> or implied by subsequent working.					
A1 dM1 A1	Correct length EM (need not be simplified) (or $BE = 24.33$) Use any trig function correctly with their values to find $\angle MEB$ Correct answer. Must be to nearest 0.1°					
(d) M1 dM1	Identify the required angle. Can be stated explicitly or implied by subsequent working.					
A1	Use any trig function correctly to obtain the size of a correct angle Correct answer. Must be to nearest 0.1°unless already penalised.					



